Lab 4 Assignment — Stochasticity and Extinction
Due before your next lab

Random Numbers in Excel

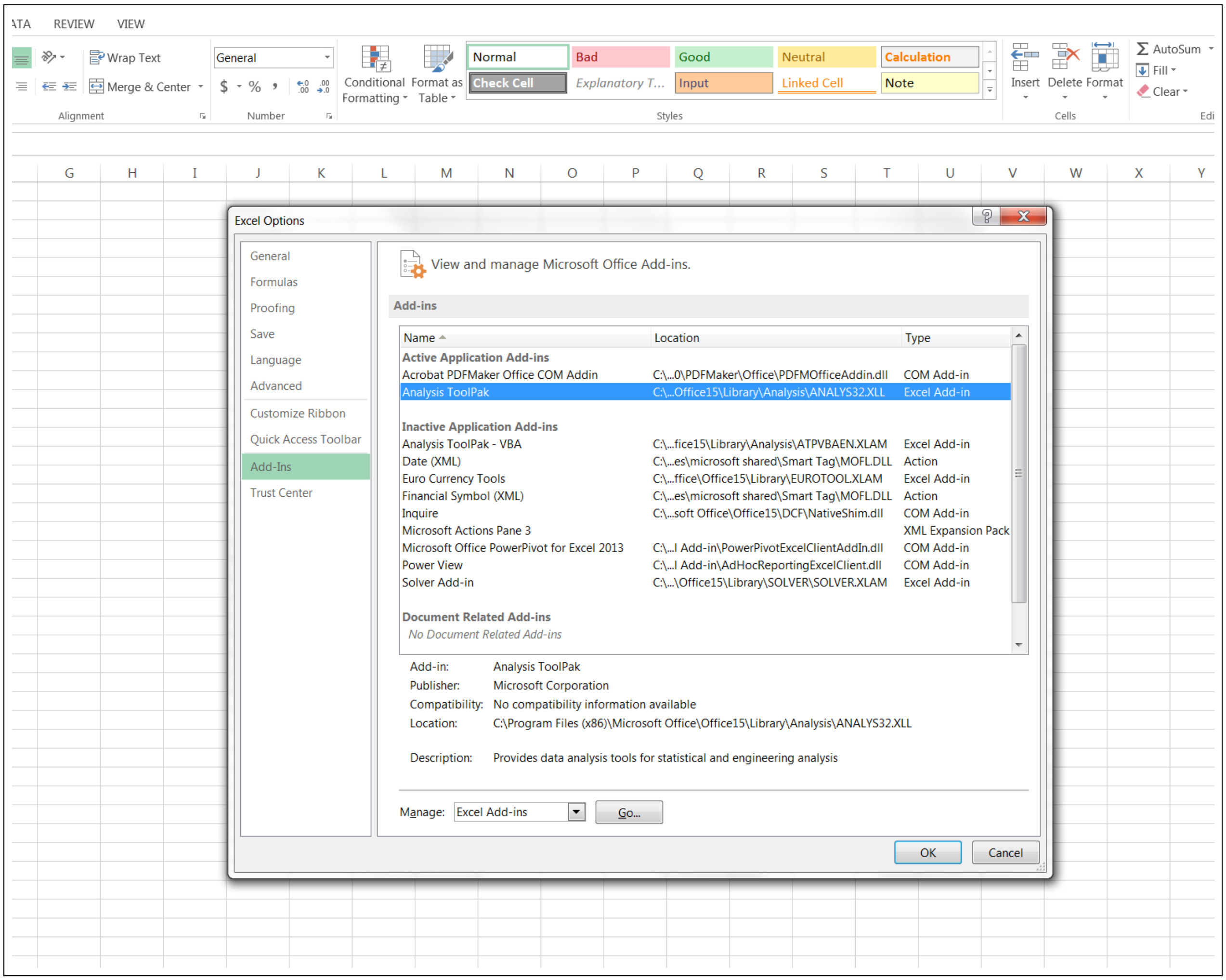


Figure 1: To generate random numbers in Excel, you must load the "Analysis Toolpak" add-in by clicking `File` > `Options` > `Add-Ins` > `Analysis Toolpak`. Then hit "Go" (not "OK") and select "Analysis Tookpak" again.

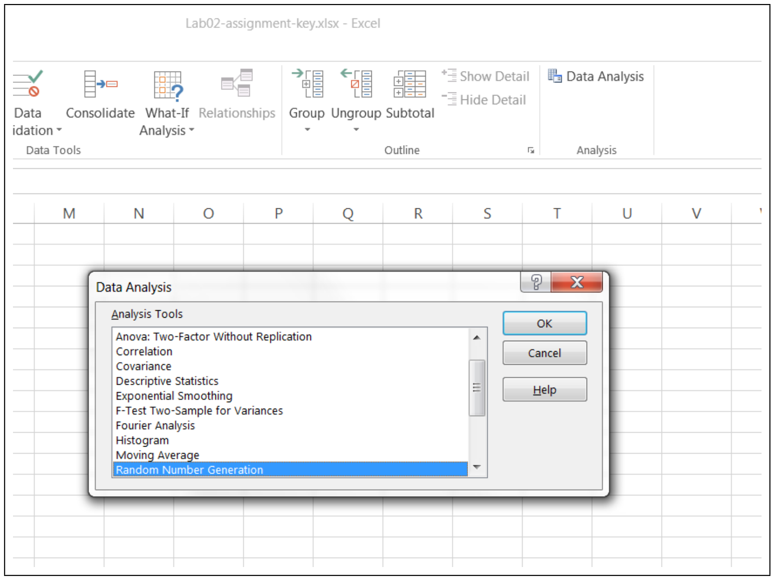


Figure 2: Once the Analysis ToolPak is installed, you can generate random numbers by clicking the "Data Analysis" button in the "Data" tab.

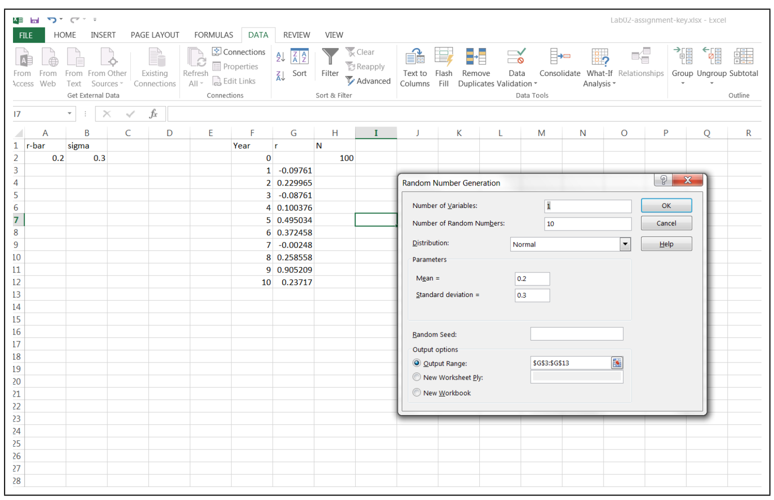


Figure 3: We will use three distributions: Normal, Binomial, and Poisson. The latter two are only used in Exercise III. Note: You can think of the "number of variables" as the number of columns of random variables, and the "number of random numbers" as the number of cells in each column.

Answer each of the following questions and upload your completed Excel file to ELC. Be sure to show your calculations.

Exercise I

1. If a population is growing geometrically and there is no random variation, what is the time to quasi-extinction (T_e) when $r = -0.2$ and $N_0 = 500$. Assume quasi-extinction occurs when N falls below 20 individuals.
2. Suppose that there is no demographic stochasticity, but environmental stochasticity occurs with $X_t \sim \text{Normal}(\mu = 0, \sigma = 10)$. Conduct 10 simulations over a 30 year time period, and force the population size to zero if it falls below the threshold. This can be done by multiplying the growth equation by a "test statement" such as `CELL>$O$2`. Plot the projections. (Hint: Generate the random variables before trying to calculate N_t).
3. What is the average T_e based on these 10 simulations?

Exercise II

1. Simulate 10 populations under a logistic growth model with N_0=20, K=40, and a normally-distributed growth rate $r_{\text{max}_t} \sim \text{Norm}(\mu = 0.1, \sigma = 1)$. Assume that the quasi-extinction threshold is equal to 10, and force the population size to zero if it falls below the threshold using a test statement as before. Plot the projections.
2. What are the quasi-extinction risks at years 5, 10, and 15?

Exercise III
In this exercise, you will be exposed to two new probably distributions: the Poisson and the binomial. The Poisson distribution is useful for data that are non-negative integers. It only has a single parameter (often called "lambda" but not to be confused with the finite rate of increase) that describes the expected value of the count data. In stochastic population models, the Poisson distribution can be used to model the number of births (B_t) that occur in a time interval.

The binomial distribution is also useful for data that are non-negative integers, but it has an upper bound. In population models, the upper bound is often population size, and we use the model to describe how many individuals die during some time period.

Assume that a population is geographically closed such that there is no immigration or emigration. The number of individuals born is a Poisson random variable:

$$B_t \sim \text{Poisson}(N_t \times b)$$

and the number that die each year is a binomial random variable:

$$D_t \sim \text{Binomial}(N_t, d)$$

Abundance is just the number that were alive plus the number that were born, minus the number that died:

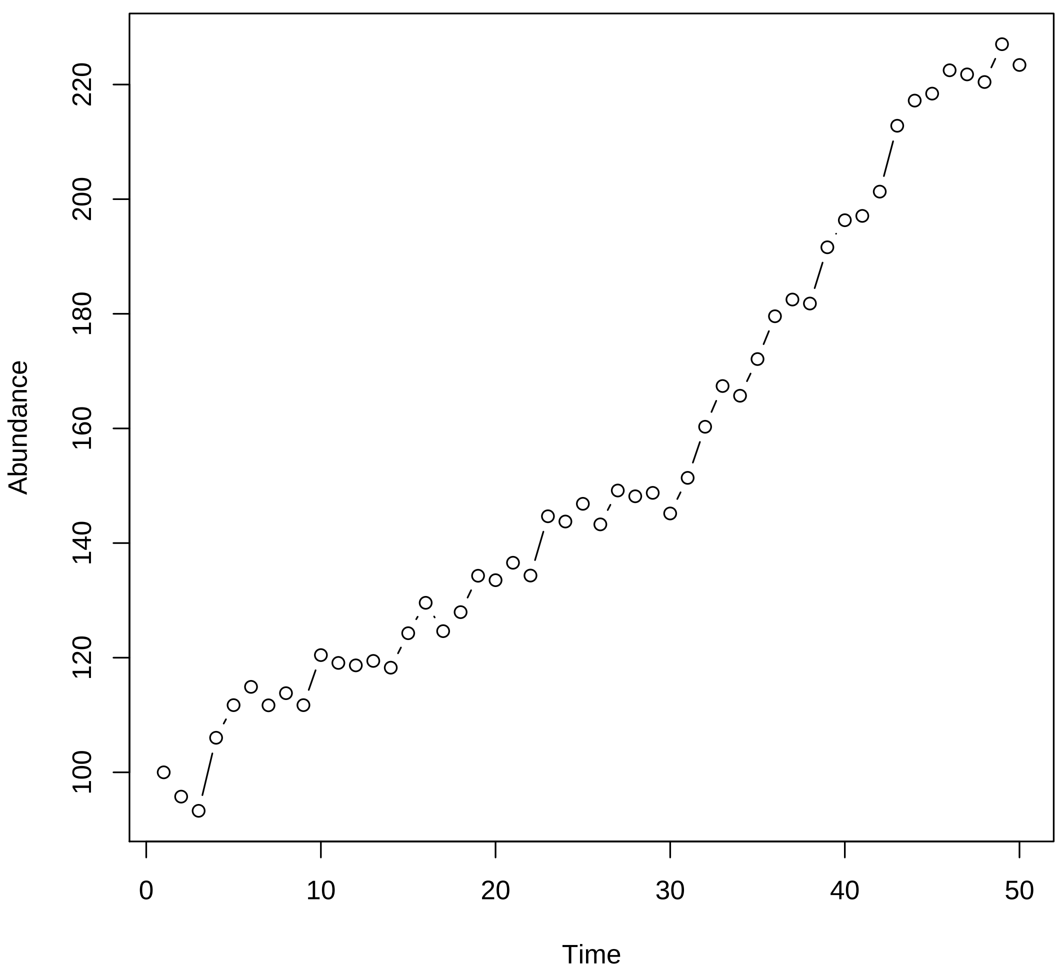
$$N_{t+1} = N_t + B_t - D_t$$

1. Beginning with $N_0 = 300$, conduct one simulation over 5 years, in which $b = 0.3$ and $d = 0.2$. Plot the results. Hint: You have to randomly generate B_t and D_t each year before you can compute N_t.
2. Do you think this population will reach a stochastic equilibrium? Why or why not? (A stochastic equilibrium occurs when a population fluctuates around a long-term average).
3. Can population size ever be less than zero under this model? Why or why not?
4. Do another simulation, but this time make the mortality rate (d) density-dependent according to the model $d_t = 0.2 + 0.001 \times Nt$. Will this population reach a stochastic equilibrium? If so, at what value of abundance does the equilibrium point occur?

Example R code

Geometric growth with environmental stochasticity

```
r <- 0.01
sigma <- 5
nYears <- 50
N1 <- rep(NA, nYears)  ## Empty vector for population size
X <- rep(NA, nYears)   ## Random variable for environmental stochasticity
N1[1] <- 100           ## Initial population size
for(t in 2:nYears) {
    X[t] <- rnorm(n=1, mean=0, sd=sigma)
    N1[t] <- N1[t-1] + N1[t-1]*r + X[t]
}
plot(1:nYears, N1, xlab="Time", ylab="Abundance", type="b")
```



Poisson-Binomial birth-death model

```r
b <- 0.1  ## Birth rate
d <- 0.2  ## Mortality rate
nYears <- 50
N2 <- rep(NA, nYears)  ## Empty vector for population size
B <- rep(NA, nYears)   ## Random variable for nBirths
D <- rep(NA, nYears)   ## Random variable for nDeaths
N2[1] <- 100           ## Initial population size
for(t in 2:nYears) {
    B[t-1] <- rpois(n=1, lambda=N2[t-1]*b)
    D[t-1] <- rbinom(n=1, size=N2[t-1], prob=d)
    N2[t] <- N2[t-1] + B[t-1] - D[t-1]
}
plot(1:nYears, N2, xlab="Time", ylab="Abundance", type="b")
```