Geometric and Exponential Growth

Today's learning objectives

The equations for geometric and exponential growth

The relationship between geometric growth and the BIDE model

The difference between continuous and discrete time models population growth

The definition of density *independent* population growth

What is Population Dynamics?

The study of spatial and temporal variation in population size and structure

Fundamental Question

How does abundance go from N_t to N_{t+1}?

Answer: The BIDE Model

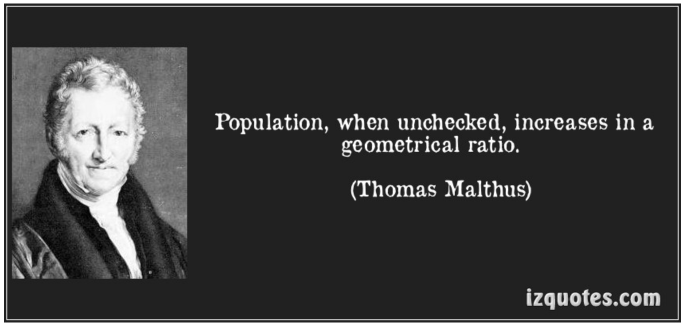
$$N_{t+1} = N_t + B_t + I_t - D_t - E_t$$

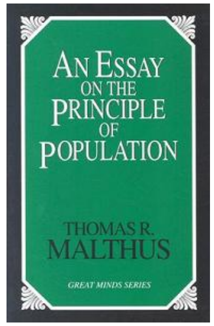
B=Births, I=Immigrations, D=Deaths, E=Emigrations

Geometric growth is a simplification of BIDE.

Exponential growth is a continuous time version of geometric growth.

Reverend Thomas Malthus, 1766–1834





Relevance To Wildlife Biology and Management

Charles Darwin (*Origin of Species*)

> *"There is no exception to the rule that every organic being increases at so high a rate, that if not destroyed, the earth would soon be covered by the progeny of a single pair."*

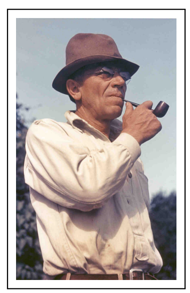
> *"Hence, as more individuals are produced than can possibly survive, there must in every case be a struggle for existence. . . "*

Aldo Leopold, Game Management 1946

"Every wild species has certain fixed habits which govern the reproductive process, and determine its maximum rate. [. . .] Thus one pair of quail, if entirely unmolested in an "ideal" environment, would increase at this rate:"

At End of	Young	Adults	Total
1st year	14	2	16
2nd year	(16/2)14=112	16	128
3rd year	(128/2)14=896	128	1024

"The maximum rate of increase is of course never attained in nature. Part of it never takes place, part of it is absorbed by natural enemies, and part of it [. . .] is absorbed by hunters."



So What Is Geometric Growth?

Discrete time, $t = 1, 2, \ldots$

$$N_t = N_0(1 + r)^t$$

Or, for one time step:

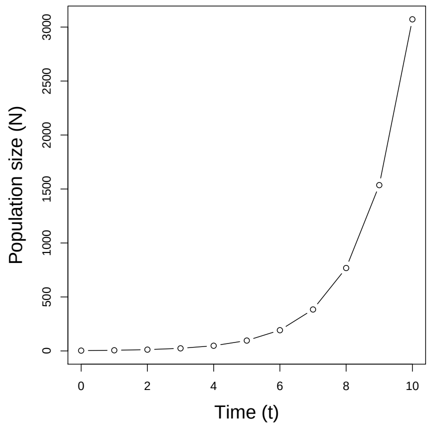
$$N_{t+1} = N_t + N_t r$$

r = discrete-time version of intrinsic rate of increase

Example, $N_{t+1} = N_t + N_t r$

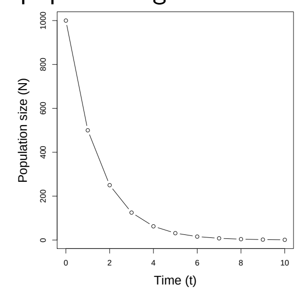
$N_0 = 3$, $r = 1$

Time (t)	Population size (N_t)
0	3
1	6
2	12
3	24
4	48
5	96
6	192
7	384
8	768
9	1536
10	3072

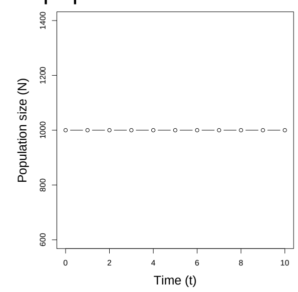


Three Possible Outcomes, $N_{t+1} = N_t + N_t r$

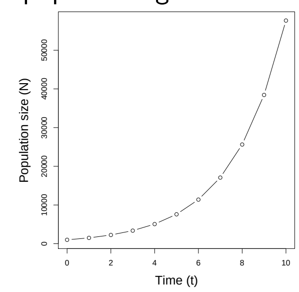
If $-1 \leq r < 0$ population goes extinct



If $r = 0$ population is stable



If $r > 0$ population grows to ∞



r and λ, $N_{t+1} = N_t + N_t r$

r is the discrete growth rate

λ is the finite growth rate

$$\lambda = \frac{N_{t+1}}{N_t}$$

$\lambda = 1 + r$

From BIDE To Geometric Growth

Fundamental equation of population ecology

$$N_{t+1} = N_t + B_t + I_t - D_t - E_t$$

N_t = Abundance at year t
B = Births
I = Immigrations
D = Deaths
E = Emigrations

From BIDE To Geometric Growth

Ignore immigration and emigration

$$N_{t+1} = N_t + B_t - D_t$$

N_t = Abundance in year t
B = Births
D = Deaths

From BIDE To Geometric Growth

Step 1: Divide both sides by N_t

$$\frac{N_{t+1}}{N_t} = 1 + \frac{B_t}{N_t} - \frac{D_t}{N_t}$$

Step 2: Write in terms of *per capita* birth and death *rates*

$$\frac{N_{t+1}}{N_t} = 1 + b - d = 1 + r = \lambda$$

Step 3: Geometric growth

$$N_{t+1} = N_t + N_t r$$

So What Is Exponential Growth?

Continuous time version of geometric growth

$$N_t = N_0 e^{rt}$$

Or, in terms of instantaneous rate of change:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN$$

N_0 = initial abundance
r = intrinsic rate of increase
t = time (any positive number)

The exponential growth model is often considered more appropriate than the geometric growth model for birth flow populations in which reproduction occurs throughout the year.

However, geometric growth models can provide a good approximation of birth flow or birth pulse populations.

Density Independent Growth

Geometric and exponential growth are examples of density independent growth

Definition: Population growth rate (r) is *not* affected by population size (N).

Implications: Resources are unlimited and there is no carrying capacity!

Model Assumptions

(1) Population is geographically closed
- No immigration
- No emigration

(2) Reproduction occurs seasonally (for geometric growth)

(3) Constant birth rate (b) and death rate (d)
- No genetic variation among individuals
- No age- or stage-structure
- No time lags

(4) No stochasticity
- No random variation in birth or death
- No random variation in environmental conditions

Can We Apply The Model To Real Data?

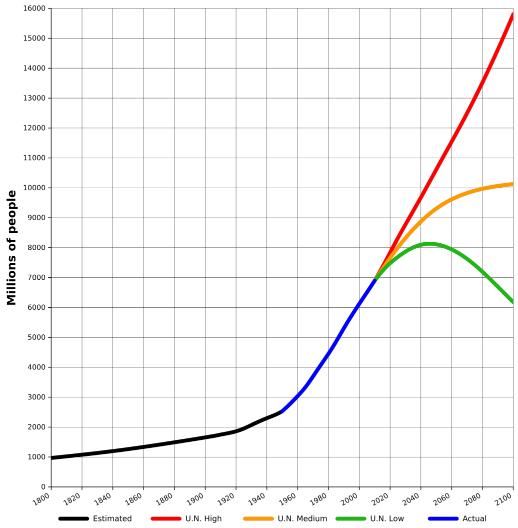
All models are wrong, but some are useful. (George Box)

Is exponential growth a useful model?

- Possibly for describing some populations during short time periods, e.g. invasive species or prey following removal of predators
- Also useful as foundation for more realistic models

Looking Ahead

Density dependence and logistic growth



Assignment

Read pages 15–19 in Conroy and Carroll

Be prepared for a quiz