Models of interspecific interactions

Predator-prey dynamics and competition

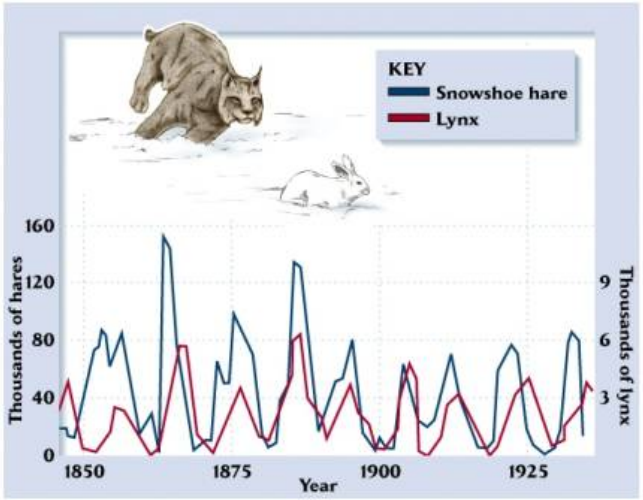
Introduction

- Lotka and Volterra developed models for both predator-prey dynamics and competitive interactions
- As usual, these models were developed as continuous-time models
- We will focus on discrete-time versions ($t = 1, 2, \ldots$)
- We will ignore potential extensions with stochasticity, age structure, spatial structure, etc. . .

Question

How should predator-prey dynamics operate?

Lynx-hare cycles



Lotka-Volterra predator-prey model

Model for prey

$$N_{t+1}^{prey} = N_t^{prey} + N_t^{prey}(r^{prey} - d^{prey} N_t^{pred})$$

Model for predator

$$N_{t+1}^{pred} = N_t^{pred} + N_t^{pred}(b^{pred} N_t^{prey} - d^{pred})$$

- Model is based on geometric growth
- r^{prey} is the growth rate of the prey in the absence of predators
- d^{prey} is the predation rate
- b^{pred} is the birth rate of the predators
- d^{pred} is the mortality rate of the predator

Equilibrium

Equilibrium for prey occurs when...

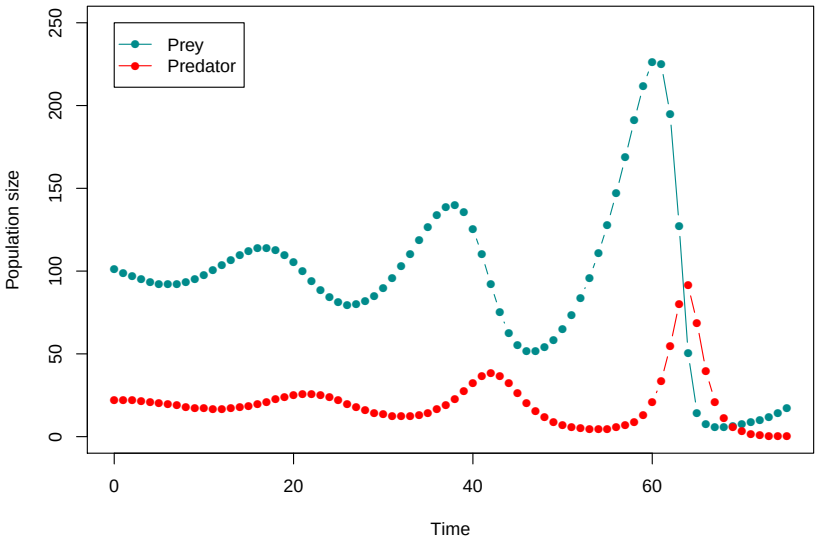
$$N^{pred} = \frac{r^{prey}}{d^{prey}}$$

Equilibrium for predators occurs when...

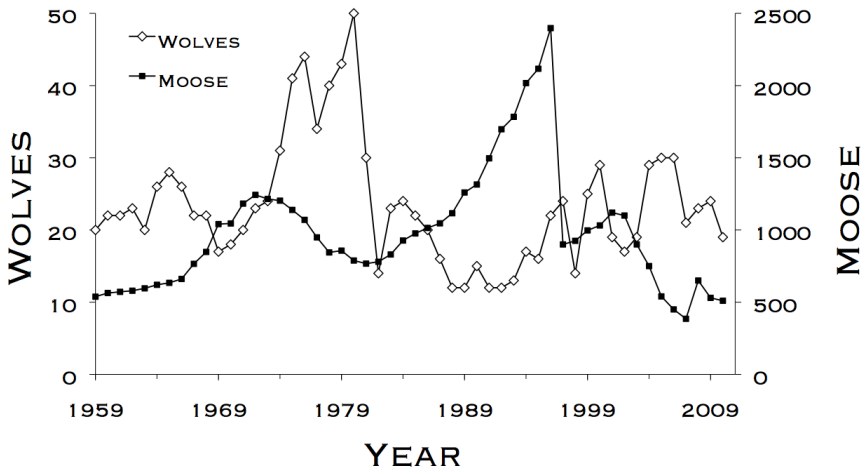
$$N^{prey} = \frac{d^{pred}}{b^{pred}}$$

However, it is rare that both equilibrium conditions will be met at the same time, and so the populations will cycle.

Model predicts population cycles

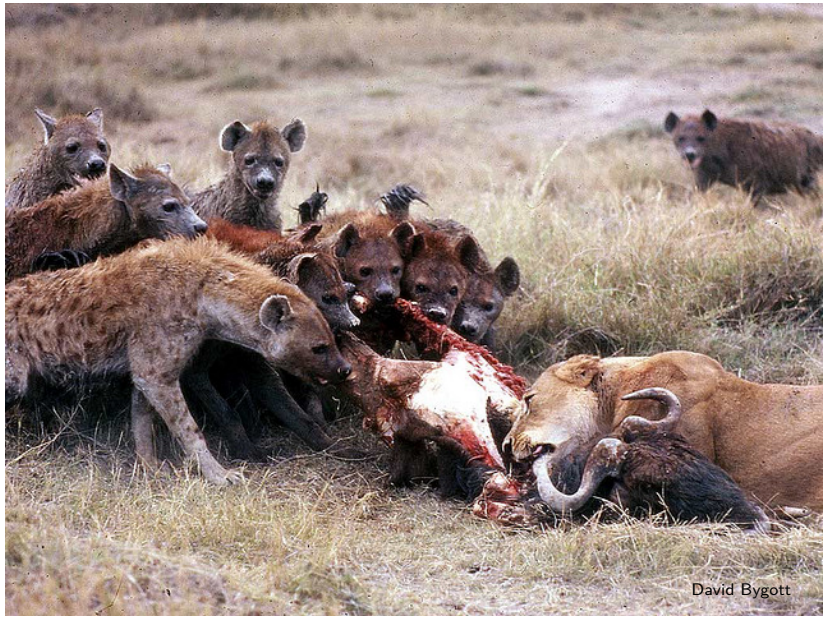


Isle Royale wolves and moose



https://youtu.be/PdwnfPurXcs

Competition



Lotka-Volterra competition model

Model for species A

$$N_{t+1}^A = N_t^A + r^A N_t^A (K^A - N_t^A - \alpha^B N_t^B)/K^A$$

Model for species B

$$N_{t+1}^B = N_t^B + r^B N_t^B (K^B - N_t^B - \alpha^A N_t^A)/K^B$$

- Model based on logistic growth
- The α parameters are competition coefficients determining how strongly each species affects the other

Equilibrium

Equilibrium for species A

$$N^A = \frac{K^A - \alpha^B K^B}{1 - \alpha^A \alpha^B}$$

Equilibrium for species B

$$N^B = \frac{K^B - \alpha^A K^A}{1 - \alpha^A \alpha^B}$$

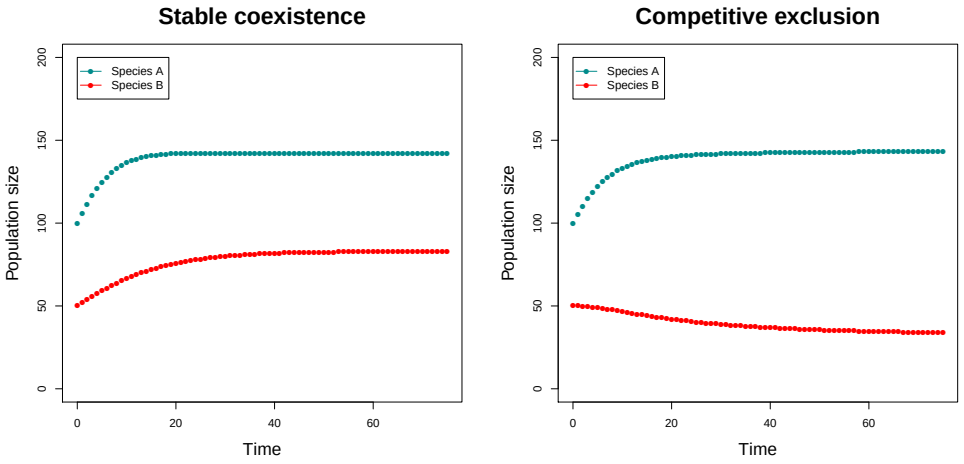
Three possible outcomes

Outcomes depend on the sign of the numerators

(1) Stable coexistence
(2) Competitive exclusion
(3) Unstable equilibrium

Competitive exclusion principle: Two species with the same niche cannot coexist on the same limiting resource

Outcomes



Summary

Predator-prey model is extension of geometric growth

- Predators and prey limit each other's growth potential

Competition model is extension of logistic growth

- Competitors influence each other's density-dependent regulation process

These models could be extended to include:

- More species
- Stochasticity
- Age structure
- Harvest
- Spatial structure
- Additional forms of density dependence