Lab 4 – Contrasts, Estimation, and Power Analysis

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FANR 6750

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Today's Topics

1. Contrasts
2. Estimation
3. Power

Chain Saw Data

```
sawData <- read.csv("sawData.csv")
sawData

##     y Brand
## 1  42     A
## 2  17     A
## 3  24     A
## 4  39     A
## 5  43     A
## 6  28     B
## 7  50     B
## 8  44     B
## 9  32     B
## 10 61     B
## 11 57     C
## 12 45     C
## 13 48     C
## 14 41     C
## 15 54     C
## 16 29     D
## 17 40     D
## 18 22     D
## 19 34     D
## 20 30     D
```


Contrasts

Suppose we want to make 3 *a priori* comparisons:

(1) Groups A&D vs B&C

(2) Groups A vs D

(3) Groups B vs C

	Comparison	Null hypothesis
1	AD vs BC	$\frac{\mu_A+\mu_D}{2} - \frac{\mu_B+\mu_C}{2} = 0$
2	A vs D	$\mu_A - \mu_D = 0$
3	B vs C	$\mu_B - \mu_C = 0$

Constructing contrasts in R

Coefficients

```
ADvBC <- c(1/2, -1/2, -1/2, 1/2)
AvD <- c(1, 0, 0, -1)
BvC <- c(0, 1, -1, 0)
```

Are they orthogonal?

```
sum(ADvBC)

## [1] 0

sum(AvD)

## [1] 0

sum(BvC)

## [1] 0
```

```
sum(ADvBC * AvD)

## [1] 0

sum(ADvBC * BvC)

## [1] 0

sum(AvD * BvC)

## [1] 0
```

Yes, they are.

We need to put the contrasts into a matrix

To use contrasts in R, each set of coefficients must be formatted as a column in a matrix.

We can use `cbind` for this:

```
contrast.mat <- cbind(ADvBC, AvD, BvC)
contrast.mat

##      ADvBC AvD BvC
## [1,]   0.5   1   0
## [2,]  -0.5   0   1
## [3,]  -0.5   0  -1
## [4,]   0.5  -1   0
```


Contrasts

Fit the model with contrasts

```
aov.out <- aov(y ~ Brand, data=sawData,
               contrasts=list(Brand=contrast.mat))
```

Now "split" apart the sum-of-squares

```
summary(aov.out, split = list(Brand =
                     list("ADvBC"=1, "AvD"=2, "BvC"=3)))

##                 Df Sum Sq Mean Sq F value  Pr(>F)
## Brand            3   1080   360.0   3.556 0.03823 *
##   Brand: ADvBC   1    980   980.0   9.679 0.00672 **
##   Brand: AvD     1     10    10.0   0.099 0.75738
##   Brand: BvC     1     90    90.0   0.889 0.35980
## Residuals       16   1620   101.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Difference in means for each contrast

Group means

```
(group.means <- tapply(sawData$y, sawData$Brand, mean))

##  A  B  C  D
## 33 43 49 31
```

Difference in means for A vs D

```
group.means <- unname(group.means) # Drop names (optional)
group.means[1] - group.means[4]

## [1] 2
```

Difference in means for B vs C

```
group.means[2] - group.means[3]

## [1] -6
```

Difference in means for AD vs BC

```
mean(group.means[c(1,4)]) - mean(group.means[2:3])

## [1] -14
```

Standard errors for each contrast

SE for A vs D

```
se.contrast(aov.out, list(sawData$Brand=="A",
                          sawData$Brand=="D"))

## [1] 6.363961
```

SE for B vs C

```
se.contrast(aov.out, list(sawData$Brand=="B",
                           sawData$Brand=="C"))

## [1] 6.363961
```

SE for AD vs BC

```
se.contrast(aov.out, list(sawData$Brand=="A" |
                          sawData$Brand=="D",
                          sawData$Brand=="B" |
                          sawData$Brand=="C"))

## [1] 4.5
```


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Estimating confidence intervals

In an ANOVA context, confidence intervals can be constructed using the equation:

$$\text{CI} = \text{Point estimate} \pm t_{\alpha/2,a(n-1)} \times \text{SE}$$

As usual, the hard part is computing the SE[1]

[1] See page 300 of Dowdy et al. for SE formulas

Confidence intervals from one-way ANOVA

SE's for the effect sizes (α's)

```
effects.SE <- model.tables(aov.out, type="effects",
                           se=TRUE)
effects.SE

## Tables of effects
##
##  Brand
## Brand
##   A   B   C   D
## -6   4  10  -8
##
## Standard errors of effects
##         Brand
##           4.5
## replic.     5
```

Confidence intervals from one-way ANOVA

Extract the α's and the SEs

```
# str(effects.SE)
alpha.i <- as.numeric(effects.SE$tables$Brand)
SE <- as.numeric(effects.SE$se)
```

Compute confidence intervals

```
tc <- qt(0.975, 4*(5-1))
lowerCI <- alpha.i - tc * SE
upperCI <- alpha.i + tc * SE
```

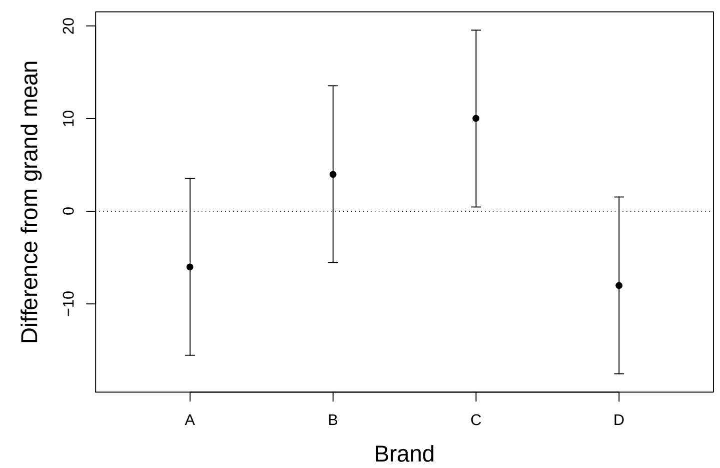
Put results into a `data.frame`

```
CI <- data.frame(effect.size=alpha.i, SE,
                 lowerCI, upperCI)
round(CI, 2)

##   effect.size  SE lowerCI upperCI
## 1          -6 4.5  -15.54    3.54
## 2           4 4.5   -5.54   13.54
## 3          10 4.5    0.46   19.54
## 4          -8 4.5  -17.54    1.54
```


Plot effects and CIs

```
plot(1:4, CI$effect.size, xlim=c(0.5, 4.5), ylim=c(-18, 20), xaxt="n",
     xlab="Brand", ylab="Difference from grand mean", pch=16, cex.lab=1.5)
axis(1, at=1:4, labels=c("A", "B", "C", "D"))
abline(h=0, lty=3)
arrows(1:4, CI$lowerCI, 1:4, CI$upperCI, code=3, angle=90, length=0.05)
```



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Power analysis for a 2-sample t-test

```
power.t.test(n=NULL, delta=3, sd=2, sig.level=0.05,
             power=0.8)

##
##      Two-sample t test power calculation
##
##               n = 8.06031
##           delta = 3
##              sd = 2
##       sig.level = 0.05
##           power = 0.8
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Power analysis for one-way ANOVA

```r
power.anova.test(groups=4, n=5, between.var=360.0,
                 within.var=101.2, power=NULL)

##
##      Balanced one-way analysis of variance power calculation
##
##          groups = 4
##               n = 5
##     between.var = 360
##      within.var = 101.2
##       sig.level = 0.05
##           power = 0.9999359
##
## NOTE: n is number in each group
```


Assignment

Researchers wish to know if food supplementation affects the growth of nestling Canada warblers. The treatment groups are: (A) No supplementation control, (B) low, (C) medium, (D) high, and (E) very high. The response variable is the weight of a 10 day old nestling.

(1) The researchers are interested in the following contrasts. Are they orthogonal?

- Groups A,B vs C,D,E
- Groups A vs B
- Groups C vs D,E
- Groups D vs E

(2) Using the `warblerWeight` data, test the null hypothesis of each contrast by constructing an ANOVA table in **R**.

(3) For each contrast:

- Compute the difference in the means
- The SE of the difference in means
- The 95% CI for the difference in means

(4) Suppose you wanted to replicate the study with a smaller sample size of $n = 2$ per treatment group? What would be your power?

Submit your answers in a self-contained script before the next lab.